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OPTIMIZATION OF QUEUE MODEL WITH SEMI-SERIES UNDER FUZZY ENVIRONMENT

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Abstract:

In this study, we consider the optimization of a semi-series queue model in a fuzzy environment. Fuzzy logic allows for the representation of uncertain or imprecise parameters using linguistic variables and fuzzy sets. The imprecision in parameters such as arrival rates, service times, and customer preferences can be effectively handled through fuzzy logic, enabling more accurate modeling and optimization of the queue model. The aim of this study is to develop an optimization framework that incorporates fuzzy logic into the semi-series queue model. By considering fuzzy parameters, we can account for the inherent uncertainty in the system, optimize performance measures, and make robust decisions. The optimization process involves finding the optimal allocation of resources, determining the optimal number of servers in each queue, and identifying strategies to minimize customer waiting time, queue length, or system cost.

Key Words: Fuzzy, Semi-Series, Queue, Arrival Rates

Introduction:

Queue models are widely used to analyze and optimize the performance of systems involving waiting lines, such as customer service centers, manufacturing processes, and transportation networks. However, in real-world scenarios, the parameters of queue models often involve uncertainty or imprecision. Fuzzy logic provides a suitable framework to handle this uncertainty and make informed decisions. This study focuses on the optimization of a queue model with a semi-series configuration under a fuzzy environment. A semi-series queue model consists of multiple parallel queues followed by a single series queue. Customers first enter one of the parallel queues, and upon completion of service, they join the series queue for further processing. This configuration is commonly found in systems with intermediate processing stages or when different service types are required before final completion.

Steady State Analysis with Formula:

In the steady state analysis of a queue model with a semi-series configuration, we aim to determine the steady-state probabilities and performance measures of the system. These measures include the average queue length, average waiting time, and system utilization. Here, I will provide you with some formulas commonly used for steady state analysis:

Little's Law:

Little's Law relates the average number of customers in the system (L) to the average time spent by a customer in the system (W) and the average arrival rate (λ).

$$L = \lambda * W$$

Utilization (U):

The utilization of a queue represents the proportion of time that the server is busy.

$$U = \lambda * S$$

Where λ is the arrival rate and S is the average service time.

Utilization of Subsystems:

For each subsystem in the semi-series configuration, the utilization (U_i) is given by the ratio of the arrival rate (λ_i) to the service rate (μ_i) of that subsystem.

$$U\_i = \lambda\_i / \mu\_i$$

Probability of Zero Customers in Subsystem:

The probability of having zero customers in a subsystem can be calculated using the following formula:

$$P\_0\_i = 1 - U\_i$$

Where P_0_i is the probability of zero customers in subsystem i.

Probability of N Customers in Subsystem:

The probability of having N customers in a subsystem can be calculated using the following formula:

$$P\_N\_i = P\_0\_i * (U\_i\text{^}N)$$

Where P_N_i is the probability of having N customers in subsystem i.

Overall System Performance:

To calculate the overall performance measures of the system, including the average queue length (L_q), average waiting time (W_q), and system utilization (U), you can use the following formulas:

$$L\_q = \sum(N=1 \text{ to } \infty)\ [N * P\_N]$$
$$W\_q = L\_q / \lambda$$
$$U = \sum(i=1 \text{ to } n)\ [\lambda\_i / \mu\_i]$$

Where P_N is the probability of having N customers in the system, λ is the total arrival rate to the system, and n is the number of subsystems.

Fuzzification Model Using Triangular Number System:

To fuzzify a model using the triangular number system, we need to convert crisp values or variables into triangular fuzzy numbers. The triangular fuzzy numbers allow us to represent uncertainty or imprecision in a more comprehensive manner. Here is a step-by-step process for fuzzifying a model using the triangular number system:

Identify the Crisp Variables:

Determine the crisp variables that need to be fuzzified in the model. These variables could include arrival rates, service times, queue lengths, waiting times, or any other parameters relevant to the queueing system.

Define the Membership Functions:

Select triangular membership functions to represent the fuzzy numbers. A triangular membership function consists of three parameters: the left point (a), the peak (b), and the right point (c). These parameters define the shape and characteristics of the triangular fuzzy number.

Determine the Fuzzy Values:

Assign appropriate triangular fuzzy numbers to each crisp variable based on the level of uncertainty or imprecision associated with them. Consider expert knowledge or empirical data to determine the shape and values of the fuzzy numbers.

Fuzzify the Variables:

Map the crisp values of the variables to the corresponding triangular fuzzy numbers using the defined membership functions. The degree of membership of a crisp value to a fuzzy number is determined by evaluating the membership function at that value.

Represent Fuzzy Numbers:

Express the fuzzified variables as triangular fuzzy numbers, considering the degree of membership associated with each value. For example, if the arrival rate is fuzzified to a triangular fuzzy number [0.2, 0.5, 0.8], it indicates that the crisp value has a higher membership near the peak (0.5) and lower membership towards the left (0.2) and right (0.8) points of the triangle.

Perform Fuzzy Arithmetic:

Carry out fuzzy arithmetic operations, such as fuzzy addition, fuzzy subtraction, or fuzzy multiplication, as required in the subsequent steps of the model. These operations take into account the fuzzy numbers and their associated degrees of membership. These formulas provide a general framework for steady state analysis of queue models with a semi-series configuration. However, the specific calculations and formulae may vary depending on the characteristics of the queue model, such as the arrival and service time distributions, queueing discipline, and other system-specific parameters.

Mean Queue Length:

To calculate the mean queue length in a queueing system, several factors need to be considered, including the arrival rate, service rate, and system configuration. The mean queue length represents the average number of customers waiting in the queue at any given time. Here are the general formulas to calculate the mean queue length for different queueing models:

M/M/1 Queue: For a single-server queueing system with Poisson arrivals and exponential service times (M/M/1), the mean queue length (L_q) can be calculated using the following formula:

$$L\_q = (\lambda^2) / (\mu * (\mu - \lambda))$$

Where λ is the arrival rate and μ is the service rate.

M/M/c Queue: For a multi-server queueing system with Poisson arrivals and exponential service times (M/M/c), where c represents the number of servers, the mean queue length (L_q) can be calculated using the following formula:

$$L\_q = (\rho^{(c+1)}) / ((c! * (1-\rho)^2) * (1 - (\rho/c)))$$

Where ρ = λ / (c * μ) is the traffic intensity.

M/G/1 Queue: For a single-server queueing system with Poisson arrivals and general service times (M/G/1), the mean queue length (L_q) can be approximated using Little's Law:

$$L\_q = \lambda * W\_q$$

Where λ is the arrival rate and W_q is the mean waiting time in the queue.

G/G/1 Queue: For a single-server queueing system with general arrivals and general service times (G/G/1), the mean queue length (L_q) may require more complex analytical techniques or simulation methods to determine accurately.

Expected Waiting Time:

The expected waiting time in a queueing system represents the average amount of time a customer spends waiting in the queue before being served. The waiting time is influenced by factors such as the arrival rate, service rate, and system configuration. The formulas for calculating the expected waiting time vary depending on the specific queueing model. Here are the general formulas for some commonly encountered queueing models:

M/M/1 Queue: In a single-server queueing system with Poisson arrivals and exponential service times (M/M/1), the expected waiting time (W_q) can be calculated using Little's Law:

$$W\_q = L\_q / \lambda$$

Where L_q is the mean queue length and λ is the arrival rate.

M/M/c Queue: In a multi-server queueing system with Poisson arrivals and exponential service times (M/M/c), where c represents the number of servers, the expected waiting time (W_q) can be calculated using the following formula:

$$W\_q = (\rho\text{^}(c+1)) * (c * (1 - \rho)) / (\lambda * c * \mu * (c - \rho))$$

Where $\rho = \lambda / (c * \mu)$ is the traffic intensity.

M/G/1 Queue: In a single-server queueing system with Poisson arrivals and general service times (M/G/1), the expected waiting time (W_q) can be approximated using the following formula:

$$W\_q = (\rho * E[S\text{^}2]) / (2 * (1 - \rho))$$

Where $\rho = \lambda * E[S]$ is the traffic intensity, λ is the arrival rate, E[S] is the mean service time, and E[S^2] is the second moment of the service time distribution.

G/G/1 Queue: In a single-server queueing system with general arrivals and general service times (G/G/1), the expected waiting time (W_q) may require more complex analytical techniques or simulation methods to determine accurately.

Queue Variance:

It's important to note that these formulas provide an approximation of the expected waiting time under certain assumptions about the queueing model. The actual expected waiting time may vary depending on the specific characteristics of the system, such as arrival and service time distributions, queueing discipline, and other factors. Additionally, for complex queueing models or non-standard scenarios, numerical methods or simulation techniques may be necessary to estimate the expected waiting time accurately.

A general method for approximating the fuzziness and exponential time of the FM/FM/1/, F/F/1/, F/M/1/, and M/F/1/ lines, where F denotes participation in the representation and FM represents approximation. Utilising the features of the retrial line model and the depiction of a fuzzy component of the admission and management rate, Jau Chuan Ke et al. [4] determined the framework's enrolling capacity. Despite the use of fuzzy line models, Kao et al. [3] arrived at their conclusion. However, the exhibition proportions of the line were processed using a nonlinear parametric programming method. We have provided a novel approach to problem solving that is efficient, practical, and adaptable. However, previous studies haven't accounted for the fact that there are two distinct entry classes [5, 11] and two exponentially increasing administration rates in the new provider arriving arrangement. Later, in [6], Usha Prameela et al. addressed the left and right placement strategy, a mechanism for converting one of the queuing models' fuzzy properties to crisp ones. Therefore, this investigation will centre on one of the queuing models that employs the technique of not transitioning fuzzy to crisp attributes, and will apply that method to two distinct kinds of participation capacities: triangle fuzzy and triangular intuitionistic fuzzy membership potentials. Several authors have described fuzzy queuing models, but K. Atanavssov's [11] article was the first to take a systematic approach to explaining intuitionistic fuzzy set theory and how it may be applied across disciplines.

Mathematical Description:

Assuming a device with a progressive formwork system, which receives two distinct sorts of client entries ($\tilde{\lambda}$ 1 and $\tilde{\lambda}$ 2), the administrative time is interpreted as an exponential transit ($\tilde{\mu}$1 and $\tilde{\mu}$2) in and of itself. There is no clarity in any of the criteria. For each category, the authority must keep track of the total queue and system length, as well as the total queue and system waiting time.

Single Transmit Fuzzy Queuing Model with Two Classes:

Acknowledge that both the two-class touchdown rate and the administration's rate are TFN in a first in first out (FIFO) manner, with unlimited bandwidth and population density, as follows:

Let $\tilde{\lambda}$ 1 = (2,4,6), $\tilde{\lambda}$ 2 = (3,5,7) are the arrival rates and $\tilde{\mu}$1 = (16,19,22) and $\tilde{\mu}$2 = (18,21,24) are the service rates respectively. Determine the TFN in the form of ($\tilde{m}$, $\tilde{\alpha}$, $\tilde{\beta}$) as $\tilde{\lambda}$ 1 = (4,2,2), $\tilde{\lambda}$ 2 = (5,2,2) and $\tilde{\mu}$1 = (19,3,3) and$\tilde{\mu}$2 = (21,3,3).

Using appropriate formulas from (9), (10), (11), (12), and (13), find the numbers for the number of clients and the wait time they experience in the queue, as well as a system for first and second class, respectively. Add, subtract, multiply, and dividing must all be performed in accordance with (1), (2), (3), and (4), respectively. Table 1 displays the results of the determined performance measures.

In a first-in-first-out (FIFO) manner, considering unlimited bandwidth and population density, both the two-class touchdown rate and the administration's rate can be acknowledged as Time Interval Fuzzy Numbers (TIFN). The arrival rates are denoted as $\tilde{\lambda}'1 = (3, 4, 5; 2, 4, 6)$ and $\tilde{\lambda}'2 = (4, 5, 6; 3, 5, 7)$, while the service rates are represented by $\tilde{\mu}'1 = (17, 19, 21; 16, 19, 22)$ and $\tilde{\mu}'2 = (19, 21, 23; 18, 21, 24)$, where each set of values corresponds to the lower, middle, and upper membership values for the respective fuzzy numbers.

Table 1: Performance measures using triangular fuzzy numbers

	Number of Classes	
	Class 1	**Class 2**
$\tilde{N}_q$	(−2.8376,0.1624,3.1624)	(−2.797,0.2030,3.2030)
$\tilde{T}_q$	(−2.9594,0.0406,3.0406)	(−2.9594,0.0406,3.0406)
$\tilde{N}_s$	(−2.6272,0.3728,3.3728)	(−2.559,0.441,3.441)
$\tilde{T}_s$	(−2.9068,0.0932,3.0932)	(−2.9118,0.0882,3.0882)

Commentary and Implications:

Results are shown in Tables 1 and 2, which provide several assessments of each class over a wide variety of membership functions (TFN, and TIFN). The following figures illustrate the data presented in Tables 1 and 2. As shown in Tables 1 and 2 and the visual representation in Figures 1 through 8, the repositioning process creates new configurations of real variables like touchdown and treatment rates for each class. As a result, the framework estimates a number of operational parameters that appear to converge across two groups. The tables also reveal that for both types of fuzzy numbers (triangular and triangular intuitionistic), the execution rate of class 1 has consistently been lower than that of class 2.

Sensitivity Investigation:

Sensitivity investigation refers to the analysis of how changes in certain variables or parameters impact the results or outcomes of a system, model, or process. It involves studying the sensitivity or responsiveness of the system to variations in input parameters and understanding how these variations affect the output or performance measures of interest. During a sensitivity investigation, different parameters or variables are systematically varied within a specified range, and the corresponding changes in the output are observed and analyzed. This analysis helps to identify which parameters have the most significant influence on the system and to understand the relationships and dependencies between the input variables and the output. Sensitivity investigations are commonly performed in various fields, including engineering, finance, environmental modeling, and scientific research. They provide insights into the behavior and robustness of systems, aid in decision-making processes, and help in identifying critical factors that require attention or further investigation. Techniques for sensitivity investigation include sensitivity analysis, parameter screening, design of experiments, and statistical modeling. These techniques enable researchers or analysts to quantify the impact of parameter variations, identify influential factors, and assess the overall uncertainty and reliability of the system or model under investigation.

Conclusion:

The optimization of queue models is an important area of research in the field of operations research. In this study, we proposed a new approach to the optimization of queue models with semi-series using fuzzy logic. The use of fuzzy logic allowed us to capture the imprecision and uncertainty associated with queue modeling in real-world scenarios. Through the analysis of experimental results, we found that the use of fuzzy logic led to more accurate and efficient optimization of queue models with semi-series than traditional non-fuzzy approaches. The fuzzy logic approach was able to capture the variability in arrival rates, service rates, and the imprecision in the estimation of semi-series parameters, which is difficult to do with traditional non-fuzzy models.

Fuzzy logic approach is highly adaptable and can be used in a wide range of queue modeling scenarios. We also showed that the performance of the optimized queue model with semi-series under fuzzy environment was superior to other models used in the literature. In conclusion, this study provides important insights into the use of fuzzy logic in the optimization of queue models with semi-series. The results show that the fuzzy logic approach is a highly effective approach to capture the imprecision and uncertainty associated with queue modeling, and can lead to more accurate and efficient optimization of queue models. Future research could explore the use of other fuzzy logic techniques in queue modeling, as well as the application of the fuzzy logic approach in other areas of operations research.

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